## **Engineering Motes**

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### **Load Distributions on Slender Delta** Wings Having Vortex Flow

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#### Nomenclature

= aspect ratio, defined as  $b^2/S$ A

b = wing span

c= local wing chord

= mean chord Ĉ

= wing section lift coefficient, defined such that  $c_I$ 

$$1 = \int_0^1 \frac{c_1 c}{\bar{c} C_L} d \frac{y}{(b/2)}$$

= wing section bending moment coefficient, defined  $c_b$ 

$$1 = \int_0^I \frac{c_b c}{\bar{c} C_B} d \frac{y}{(b/2)}$$

and based on the wing area S and the wing span b

 $C_B$ = bending moment coefficient calculated about the wingroot

= drag coefficient

 $C_D$ 

= lift coefficient

 $C_M$ = pitching moment coefficient

 $\Delta C_p$ = lifting pressure coefficient

M = Mach number

Re = Reynolds number

S = wing area

x = chordwise body axis coordinate

= spanwise body axis coordinate y

z = body axis coordinate normal to wing

α = angle of attack

= wing leading-edge sweep Λ

#### Introduction

S UPERSONIC aircraft with thin, highly swept wings develop separation-induced vortex flow at the wing leading edge over a significant portion of their flight envelope. This separated flow reattaches inboard of the leading edge because of the roll-up of the shed vortex sheet into a pair of strong vortices located above the wing. Because of the low pressures associated with these vortices, additional lift is generated on the wing.

Because of the nature of this type of flow, potential-flow theories which assume attached flow do not predict accurately either the distribution or magnitude of the wing aerodynamic load. 1,2 Early theories 3 which include the shed vortices, but which make slender-body conical-flow assumptions, are also inaccurate. The suction analogy method developed at the NASA-Langley Research Center 1,4,5 circumvents these problems and provides much improved predictions of  $C_L$ ,  $C_D$ , and  $C_M$ , but does not predict the detailed surface load

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distribution. Since many of the critical wing structural loads for maneuvering of supersonic cruise aircraft occur when this separation-induced vortex flow exists, the Langley Research Center has been involved in the development of methods for accurately predicting the surface load distribution for this type of flow. As part of this program, a free vortex sheet nonconical flow lifting-surface method is being developed by the Boeing Company under contract with the Langley Research Center. 6

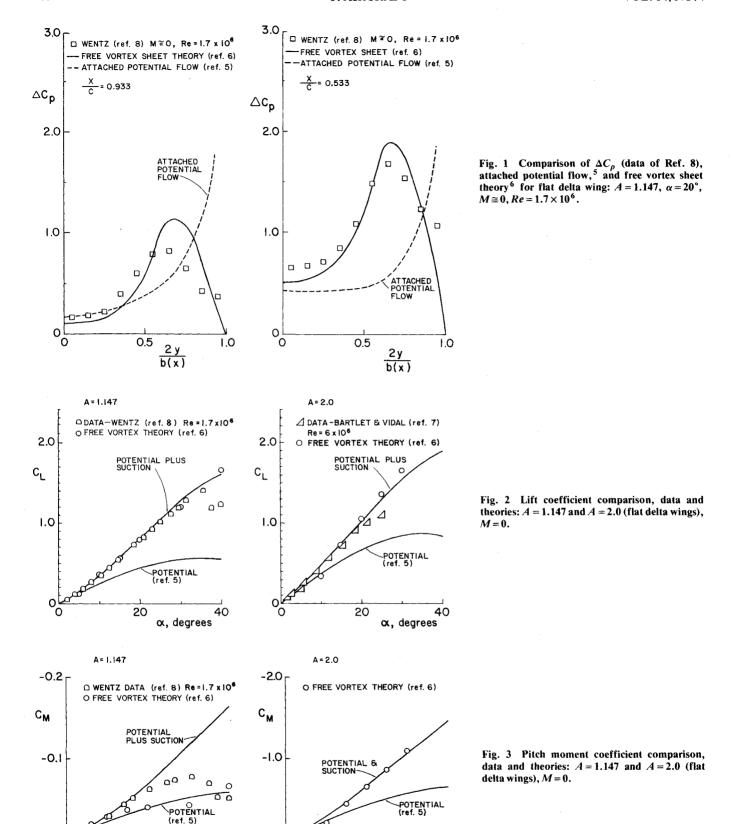
This Note compares some results obtained by application of the initial form of this theory for incompressible flow with attached-flow theory, and with available experimental data for sharp-edged delta wings<sup>7,8</sup> to determine the accuracy of the method. Also, the error in predicted structural loads using attached-flow theory is investigated. Comparisons are made of force and moment coefficients and load distributions. Although the suction analogy does not predict the surface load distribution, an estimate of the root bending moment can be made by assuming the vortex lift to be concentrated at the wing leading edge. Although this assumption can be anticipated to be in error at large angles of attack due to the inboard movement of the center of vortex lift, it also is compared with experiment for interest, since it might be considered to provide an "upper bound" in evaluating the results of other theories.

#### Results

An example of the predicted pressure coefficient distribution for the free vortex sheet method<sup>6</sup> is compared with experimental pressure measurements by Wentz<sup>8</sup> in Fig. 1. The solution is for a flat, sharp-edged delta wing of aspect ratio A = 1.147. Angle of attack is nominally  $\alpha = 20^{\circ}$ . Agreement between data and theoretical results of this method is qualitatively good with some apparent overprediction of  $\Delta C_p$  near the apex. However, the data in this region are sparse and also may have been averaged appreciably over the pressure tap diameter. Load distribution for an attached-flow theory also is displayed and illustrates the large errors that can occur if attached-flow theory is used for structural design loads.

As part of a more quantitative comparison in Fig. 2, lift coefficient vs angle of attack is displayed for A = 1.147 and A= 2.0 - flat, sharp-edged delta wings. Experimental force data 7,8 are compared with theoretical predictions of attached flow, 2 suction analogy, 5 and the free vortex sheet method. 6 The attached-flow method is included for reference and illustrates the gross underprediction of the lift capability. Both the suction analogy and free vortex sheet method accurately predict  $C_L$  over a wide range of angle of attack, with some overprediction above values of  $\alpha$  where vortex bursting occurs ahead of the wing trailing edge. Note that at  $\alpha = 10^{\circ}$ the free vortex sheet method yields a poor  $C_L$  value; this is perhaps because of the sparseness of the wing panelling in the current computer code.

Figure 3 compares pitching moment coefficient for the same three theories with data from Refs. 7 and 8 for the same delta wing configurations discussed in Fig. 2. As in Fig. 2, attached-flow theory underpredicts the aerodynamic coefficient. The moment axes are located at the wing apex for A = 2.0, and at the wing aerodynamic center for A = 1.147. The suction analogy overpredicts  $C_M$  for  $\alpha > 17^\circ$ , whereas the free vortex method underpredicts  $C_M$  for  $\alpha > 10^\circ$  compared to data for A = 1.147. Note the expanded scale for  $C_M$  for the A= 1.147 wing.



Bending moment coefficients calculated about the wingroot are presented in Fig. 4, again for A = 1.147 and 2.0 -flat deltas. Attached flow again underpredicts  $C_B$  compared to calculations using pressure data at A = 1.147. The suction analogy predicts significantly higher bending moments than the free vortex sheet method for both aspect ratios. For A = 1.147, the free vortex sheet method is seen to predict  $C_B$ 

a, degrees

a, degrees

accurately for  $\alpha \le 30^\circ$ . In Fig. 5 values of  $C_B$ , as predicted by the three theoretical methods, are compared at  $\alpha = 20^\circ$  for an aspect ratio range of from 0.5 to 2.0. Over this range, attached flow underpredicts  $C_B$  relative to the data, whereas the suction analogy overpredicts  $C_B$ . Again, note that at A = 1.147, it is the free vortex method which accurately predicts the bending moment.

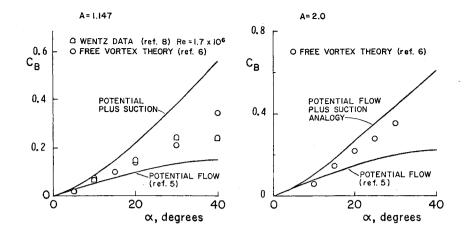


Fig. 4 Wing Root bending coefficient comparison, data and theories: A = 1.147 and A = 2.0 (flat delta wings), M = 0.

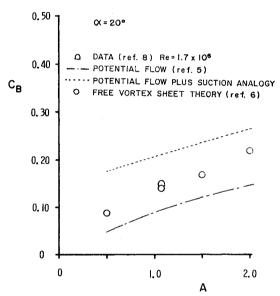


Fig. 5 Wing Root bending moment comparison for flat delta wing at  $\alpha = 20^{\circ}$ , aspect ratio varying, M = 0.

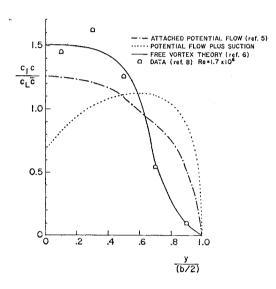
In Fig. 6 the span loading,  $c_1c/C_L\bar{c}$ , is displayed at  $\alpha=20^\circ$ , A=1.147 as calculated using pressure data of Ref. 8 and the three theoretical methods. Span loading calculated using the free vortex theory agrees best with the data. Both attached flow and the suction analogy overpredict span loading near the wingtip.

Also in Fig. 6, a spanwise bending moment coefficient, defined in a fashion analogous to the span loading, is displayed for A = 1.147,  $\alpha = 20^{\circ}$ . It is seen that attached flow underpredicts  $C_B$ , the suction analogy overpredicts  $C_B$ , whereas the free vortex theory accurately predicts both magnitude and spanwise distribution of  $C_B$ .

#### Discussion

Attached-flow theory gives load distributions which are clearly inadequate, underpredicting the values of  $C_L$ ,  $C_M$ , and  $C_B$ . The suction analogy yields accurate estimations of  $C_L$  and  $C_M$  while overpredicting the bending coefficient  $C_B$ . That this method gives values of  $C_B$  which are too large is explained by noting that in applying the suction analogy to  $C_B$  it was assumed that the vortex lift was concentrated along the leading edge since the analogy does not predict the surface load distribution. The actual vortices influence wing surface pressures over a finite region inboard of the wing leading edge where the bending moment arm is less.

The free vortex sheet method of Ref. 6 is seen to predict best magnitudes of  $C_L$ ,  $C_M$ , and  $C_B$  as well as the span



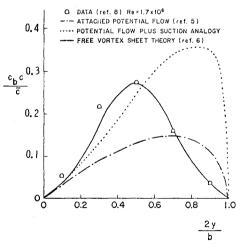


Fig. 6 Spanwise load distribution and bending moment distribution for A = 1.147 (flat delta),  $\alpha = 20^{\circ}$ , M = 0.

loading distribution and spanwise distribution of bending moment, when compared with existing data at A = 1.147 and 2.0. Thus, this theory holds much promise for the prediction of aerodynamic loading of delta wings. However, two improvements to the current computer code are needed. Wing panelling must be made finer. It is hoped that this will lessen convergence problems at high angle of attack or high aspect ratio. Also, the modelling of the shed vortex sheet must be

improved to allow investigation of more complex planforms such as planforms with streamwise side edges. Use of the conservative estimate of  $C_B$  and load distribution calculated using the suction analogy may be justifiable in applications where the additional structural weight penalty is not severe.

#### Acknowledgment

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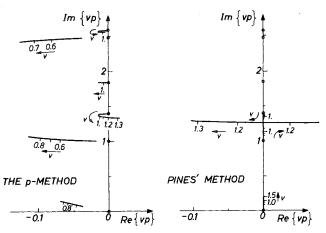
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# Test of Pines' Approximate Method in a Flutter Calculation for the Viggen Aircraft

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INES¹ and Landahl² emphasized long ago that flutter is often associated with a loss of resultant stiffness. Using this observation as a basis, Pines¹ proposed an approximate method for calculation of the flutter speed. This method was tested in a large number of cases by Ferman³, and Pines and Newman⁴ concluded that the method was "...quite accurate for primary surfaces and reasonably acceptable for control surfaces." By this Note we wish to show that the method seems to be useful also for a canard configuration such as that of the Viggen aircraft.

The approximate method has been applied to the Viggen configuration and the result is compared here to that of a corresponding flutter calculation by the p-method. In both calculations, a linear combination of five symmetric natural modes plus the rigid translation and pitch modes was used. The deflection in the natural modes mainly consists of wing bending, body bending, engine motion, wing torsion, and wing motion in the wing plane. These modes, the mass matrix,



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Fig. 1 Root loci in the complex vp plane for a Mach number of 0.7.

and the stiffness matrix were determined by means of data from a ground vibration test, and the aerodynamic matrix was calculated by the Polar Coordinate Method  $^5$  for some values of the reduced frequency k. An approximation to the aerodynamic matrix then was determined in the form of a polynomial in ik with real coefficient matrices. This was used in the flutter calculation by the p-method, whereas only the zero-order coefficient matrix was used in the calculation by the approximate method.

Let U denote the speed of flight, L a reference length,  $\omega_n$  the frequency of the nth natural mode, and  $p=\mu+ik$  the dimensionless complex frequency parameter that appears upon Laplace transformation of the equations of motion with respect to the dimensionless time  $\tau = Ut/L$ . The eigenvalues  $p=p_n$ , which are roots of the characteristic equation and vary with the normalized speed  $v=U/(\omega_1 L)$ , form loci in the complex p-plane. We prefer, however, to consider the corresponding values of the product  $vp=(U/\omega_1 L)$  (p'L/U) =  $p'/\omega_1$  and the loci of these in the vp plane. The values of vp for v=0 represent starting-points for the loci. If the virtual masses are small, the moduli of the starting points which lie on the imaginary axis are approximately equal to the frequency ratios  $\omega_n/\omega_1$ .

The results of the calculations by the *p*-method and the approximate method are illustrated by the two diagrams in Fig. 1. It is seen that it is the wing bending mode and the body bending mode which couple and form the aeroelastic mode that goes unstable. For an increasing subcritical speed, the roots obtained by the approximate method for these two modes lie on the imaginary axis and move toward each other until they meet. This occurs at a speed which, according to Pines, may be considered an approximation to the flutter speed. For still higher speeds, the roots leave the imaginary axis in opposite directions.

The corresponding loci in the diagram obtained by the p-method are to some extent similar. However, those parts of the loci which correspond to subcritical speeds lie to the left of the imaginary axis and the roots do not reach each other. They move also in this case essentially in opposite directions away from each other as soon as a speed slightly less than the flutter speed has been exceeded.

From the loci produced by the p-method and the approximate method we may read the values 1.20 and 1.15, respectively, for the flutter speed. The latter is only 4% lower than the former. We therefore conclude that the approximate method is useful also for a canard configuration.

We may add that a satisfactory flutter margin exists. The flutter speed result v = 1.2, which applies to standard day sealevel density and zero angle of incidence, implies that the flutter speed at a flight Mach number of 0.7 is twice as large as the corresponding flight speed at standard day sea-level temperature.

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